







Generalized Contingency Analysis Based on Graph Theory and Line Outage Distribution Factor

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Abstract—Identifying the multiple critical components in power systems whose absence together has severe impact on system performance is a crucial problem for power systems known as $(N - x)$ contingency analysis. However, the inherent combinatorial feature of the $N - x$ contingency analysis problem incurs by the increase of x in the $(N - x)$ term, making the problem intractable for even relatively small test systems. We present a new framework for identifying the $N - x$ contingencies that captures both topology and physics of the network. Graph theory provides many ways to measure power grid graphs, i.e., buses as nodes and lines as edges, allowing researchers to characterize system structure and optimize algorithms. This article proposes a scalable approach based on the group betweenness centrality concept that measures the impact of multiple components in the electric power grid as well as line outage distribution factors that find the lines whose loss has the highest impact on the power flow in the network. The proposed approach is a quick and efficient solution for identifying the most critical lines in power networks. The proposed approach is validated using various test cases, and results show that the proposed approach is able to quickly identify multiple contingencies that result in violations.

Index Terms—Betweenness centrality, contingency analysis, graph theory, line outage distribution factors.

I. INTRODUCTION

A. Motivation

INCREASING demand for electric power has forced power systems to operate closer to their limits, which in turn jeopardizes power system security by making these systems more susceptible to cascading failures [1]. Hence, this article is motivated by extreme events in electric power systems that are caused by multiple contingencies. Robust operation of the power grid requires anticipation of unplanned component outages that could trigger extreme events [2]. Electric power grids today are

already designed to be resilient against any single contingency, called $N - 1$ operational reliability. However, loss of multiple components is still a major concern that can lead to large-scale problems including system instability, uncontrolled separation, cascading outages, and voltage collapse [2]. Additionally, smart grid deployment can expose the electric power grid to purposeful and malicious attacks, where such exposure raises concerns about the possibility of malicious $N - x$ scenarios. Thus, it is important to make the system secure, not only for any given $N - 1$ contingency, but also for selected $N - x$ contingencies. This results in a large number of contingencies that need to be analyzed. Analyzing the large number of potential contingencies presents a major challenge since the computational complexity of $N - x$ contingency analysis drastically increases with the number of components outaged [3], [4]. Determining and evaluating all possible combinations of component failures is a combinatorial problem, which is not tractable even for medium-sized power systems. The problem is easily demonstrated by direct calculation of the number of combinations required for $N - 3$ and $N - 4$ contingency analysis of a 10 000-component system, which involve approximately $10^{12}/6$ and $10^{16}/24$ combinations, respectively [5].

B. Literature Review

Numerous efforts have been made on identifying the most critical components in electric power grids. Critical components have been proposed to be identified through network structural analysis. Graph-based methodologies provide promising approaches for finding these critical components, e.g., lines, buses, in power systems [6]. Graph topology can also be used to detect anomalies in electric power grids [7]. Developed in the context of social network analysis, betweenness centrality is a concept that captures the relative importance of an entity in a network [8]. Betweenness centrality reflects the edge or node importance in network structure [9].

Multiple studies utilize the betweenness centrality concept to identify the most critical components in power grids [6], [10]. The proposed approach in [6] applies network centrality measures to fulfill the $N - 1$ contingency analysis. Similarly, the approach in [10] uses different centrality measures, including node and edge betweenness centrality, to identify important nodes and edges in power systems. A graph edge betweenness centrality measure is proposed in [11] to perform contingency analysis of large scale power grids. The betweenness centrality

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concept is extended in [12] to account for $N - x$ contingency selection for $x \geq 2$. The approaches in [11] and [12] leverage the betweenness centrality concept to find the critical lines in power network without taking the engineering features of the system into account. Although these approaches can find the critical lines from the graph theory prospective but not addressing the network's physics causes these algorithm to miss critical lines in the system that their outages might trigger sever violations in the system.

The physics of the electric power grid cannot be addressed by pure structural analysis such as centrality metric application. Thus, it is essential to have a more comprehensive model for approximating the failure behavior of electric power grids [13]. To address these issues, different properties of electric power grids along with the betweenness centrality concept have been taken into consideration to address the physics of the networks. Electrical centrality is a metric that is calculated based on the system impedance matrix Z^{bus} and utilizes the centrality metric, to explain why in electric power grids a few highly-connected bus failures are able to cause a cascading effect, and it is investigated in different studies [14]–[16]. The betweenness centrality concept is applied in [17] to the graph of electric power grid, weighted by the corresponding admittance matrix, for vulnerability analysis of power network. Maximal load demand and the capacity of generators are considered along with the betweenness centrality in [18] to analyze the vulnerability of electric power grid. Electrical distance, an essential feature of power network that can be interpreted as the difficulty or cost in transmitting physical quantities between any pair of vertices, is taken into account in the betweenness centrality metric in [19] to capture the grid properties. The electrical distance between generation bus i and load bus j is defined as the equivalent impedance Z_{ij} [20], which considers the impedance of transmission lines between these buses. Although these studies consider the electrical properties of electric power grids, none of them captures the impacts that loss of a component might have on the system.

C. Contributions and Paper Organization

Although structural analysis of electric power grids provides useful information for vulnerability analysis, it is necessary to capture electrical characteristics that measure impact of component loss to identify important components for contingency analysis. To address this issue, we leverage line outage distribution factors (LODFs) [21], a sensitivity metric of how a change in a line's status affects the flows on other lines in the system, to capture the physics of the power network and, thus, to improve the accuracy of the results. The LODF metric has already been used in identifying multiple contingencies in power systems [21], but the approach proposed in [21] is limited to $N - 2$ contingency analysis. Additionally, in this article, the betweenness centrality factor is extended to group betweenness centrality, which facilitates searching for multiple critical components in the network.

In this article, we propose a tractable generalized contingency selection approach based on the graph theory concept of group

betweenness centrality together with line outage distribution factors that identifies groups of components whose loss would have severe impact on the power system. We hereafter refer to those components as critical components. Existing literature does not provide a computationally tractable and accurate solution methodology for performing $N - x$ contingency analysis in power systems. This article bridges this gap by proposing a graph-based contingency analysis capable of performing $N - x$ contingency analysis on large test systems, irrespective of their size. The proposed approach is a generalization of the contingency analysis application because it can identify multiple critical lines in power systems, without limitation on system size N or number of components x . The term “generalized” means that the method is designed to be applied as a general solution to multiple-element contingency analysis, beyond $N - 1$ or $N - 2$. The proposed method is demonstrated for line outages; without loss of generality, it can also be used in a similar way for node outages. Moreover, augmenting the LODF metric to the betweenness centrality concept acts as a filter that enables the proposed method to quickly identify the most critical lines.

The contribution of this article is threefold. First, we extend the betweenness centrality concept to the group betweenness centrality to perform $N - x$ contingency analysis. Second, we leverage LODFs to facilitate the proposed algorithm to augment line outage impact on power flows. Third, we validate that the proposed method is computationally tractable for multiple $N - x$ contingency analysis in large power systems with a couple of thousands of lines.

This article is organized as follows. Section II reviews the betweenness centrality measure and its extension, i.e., group betweenness centrality. Section III describes the proposed generalized contingency analysis in finding the most critical lines in the network. Section IV empirically evaluates the proposed approach. Section VI presents the discussion and achievement. Finally, Section VII concludes this article.

II. CENTRALITY DEFINITION AND EXTENSION

Centrality metrics are used in network science to rank the relative importance of vertices and edges in a graph. In network analysis, there are several metrics for the centrality of a vertex or an edge [10]. The generalized contingency analysis method proposed in this article uses group betweenness centrality of the edges to find the most critical lines in power systems. In this section, we first review the definition of betweenness centrality, and then we present its extension to group betweenness centrality.

A. Betweenness Centrality

Electric power grids can be modeled as graphs $G := (V, E)$, where V (vertices) and E (edges) are the sets of buses and lines in the system. Betweenness centrality measures the extent to which a vertex lies on paths between other vertices. Vertices with high betweenness may have considerable influence within a network since more paths that connect different vertices pass through them. They are also the ones whose removal from the network will most likely disrupt information or power flows between other vertices because they lie on the largest number of

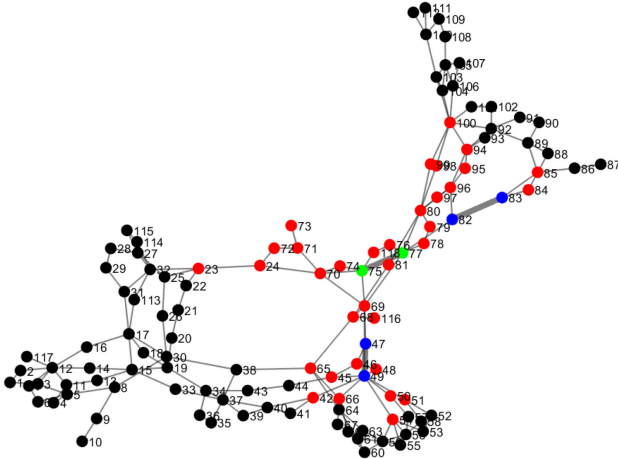


Fig. 1. Equivalent graph of the IEEE 118-bus test case. The green nodes show both ends of the line whose outage has the highest LODF factor. All the neighbors with a three-hop distance (i.e., search level equals three) from the green nodes are shown by red nodes. The lines with the blue end-nodes have the second and third highest LODF values. These lines are used to implement the concept of group betweenness centrality for the $N - 3$ contingency analysis. The set of red nodes are investigated to determine the critical lines.

paths connecting different vertices. The betweenness centrality concept can be extended from vertices to measure the influence of edges in a graph. Betweenness centrality is defined as the ratio of the number of shortest paths that pass through an edge to the total number of shortest paths between all possible pairs of vertices. Fig. 1 visualizes the concepts of the betweenness centrality and its expansion, group betweenness centrality, to make them more understandable. Mathematically, betweenness centrality of an edge can be expressed as

$$BC(e) = \sum_{s,t \in V} \frac{\sigma(s,t|e)}{\sigma(s,t)} \quad (1)$$

where $\sigma(s,t|e)$ is the number of shortest paths in the graph between s and t that contain edge e , and $\sigma(s,t)$ represents the number of shortest paths in the graph between s and t .

The betweenness centrality concept measures the importance of a single node (edge) and, thus, can be applied to $N - 1$ contingency analysis. An underlying assumption for the proposed $N - x$ contingency analysis is that cascading failures might occur from the simultaneous failure of x critical lines. Thus, the betweenness centrality concept needs to be adapted to measure the importance of multiple nodes (edges). In this connection, we propose a new group betweenness centrality that is explained in sequel.

B. Group Betweenness Centrality

The goal of the group betweenness centrality is to identify a set of the most important components whose loss has a severe impact on the network. However, the betweenness centrality metric is for an individual component (a vertex or an edge). Thus, to perform the $N - x$ contingency analysis, we extend the

betweenness centrality metric to consider a group of components

$$GBC(E_G^i) = \sum_{s,t \in V \setminus E_G^i} \frac{\sigma(s,t|E_G^i)}{\sigma(s,t)} \quad (2)$$

where E_G^i is a subset of edges of interest, $\sigma(s,t)$ is the number of shortest paths between s and t , and $\sigma(s,t|E_G^i)$ is the number of shortest paths between s and t that contain any element in E_G^i . The notion of group betweenness centrality was first introduced in [22] and [23] to identify groups of individuals who have collective influence in a social network.

For implementing the group betweenness centrality in (2), we first need to determine groups of lines, with the same number of lines (i.e., x), that need to be evaluated. However, the number of groups that need to be evaluated increases exponentially with respect to the number of elements in the group $\binom{N}{x}$, which makes the group betweenness centrality in (2) computationally intractable for large test cases. To cope with this issue, we leverage the line outage distribution factors, explained in Section III, to select limited numbers of groups out of the astronomical group of components that needs to be evaluated. Leveraging LODFs to select groups of lines whose loss has severe impact on transmission power systems drastically reduces the number of groups that need to be evaluated; this is elaborated in Section IV.

III. SOLUTION METHODOLOGY

This section leverages the group betweenness centrality concept presented in Section II-B and the LODF metric to identify the most critical lines in electric power grids. The pure graph theory information, i.e., group betweenness centrality, cannot fully address the characteristic of the electric power grid. Thus, it is essential to incorporate the LODF metric to take the electric power grid's features into account. The next section reviews the LODF metric first and then presents the proposed algorithm that leverages LODFs and group betweenness centrality to select critical $N - x$ contingencies.

A. Line Outage Distribution Factors

LODFs are a sensitivity measure of how a change in a line's status affects the flows on other lines in the system. LODFs are used extensively when modeling the linear impact of contingencies such as in PowerWorld Simulator [24]. LODF metrics in electric power grids provide approximate but quick solutions to estimate the change in the line flows. The quick computation of the LODF metric makes it an attractive measure for solving different problems in power systems. The LODF metric is used to screen multiple element contingencies in [21]. LODFs are also used for detecting island formation in power networks [25], to solve the security constrained unit commitment problem [26], and for evaluating network expansion options [27].

LODFs vary with change in the topology, when an outage occurs [28]. Multiple efforts have studied calculating LODFs dynamically, i.e., after occurring outages, as well as extending the LODF definition for multiple contingency analysis [28]–[30]. Although the LODF metric changes after outages, these changes usually happen gradually as the number of outages

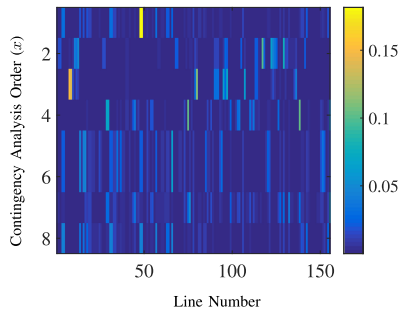


Fig. 2. Normalized LODF for lines in the system for different orders of contingency analysis (i.e., different x in the $N - x$ term) for the 200-bus test case. The colors demonstrate the normalized LODF values.

increase. Therefore, it would be a reasonable assumption for the LODF metric to remain fixed for a few line outages.

To validate this assumption, we investigate the LODF changes for different $N - x$ contingency analyses in the 200-bus test system, which are illustrated in Fig. 2. The x - and y -axis show the line number and the contingency analysis order (i.e., x in the $N - x$ term), respectively. The colors demonstrate the normalized LODF values (i.e., LODF values divided by maximum value of LODFs after each outage). Lines with very small LODFs are mitigated for illustration purposes. Fig. 2 shows that the LODF would not change drastically after a few line outages, which validates our assumption.

B. Proposed Methodology

We leverage the group betweenness centrality concept and the LODF metric to identify the most critical lines in contingency analysis. The group betweenness centrality has previously been proposed to find critical lines in a power system [12]. However, the procedure in [12] uses pure topological information to find critical lines but neglects the electrical characteristics. Moreover, the procedure utilized to identify the groups is complicated. These two shortcomings are addressed in this article via incorporating the LODF metric into our GBC methodology. To this end, we first find the LODF metric for all lines and then select the limited number of lines based on the measure in (3). To further capture the physics of power networks and prevent selecting lines that carry a small portion of power flow, the power flows in lines are incorporated in (3).

The mean of the remaining lines' LODFs seems to be an appropriate measure for determining the importance of the lost line in the system. However, the value of each LODF can be either positive or negative, so the mean of the corresponding LODFs may not capture the importance due to the offset of positive and negative LODF values for different lines. Therefore, in this article, we use the absolute value of LODFs to capture the line importance in power systems. Furthermore, losing a line might change the LODF values of a small set of lines drastically while changing all others only slightly. We are looking for a line whose loss increases the LODF of all other lines, not only a small set of them. To this end, the standard deviation of the absolute LODF values is incorporated in the proposed

metric for identifying the high impacted lines in the system. Mathematically, the proposed metric can be expressed as

$$\text{NLODF}(i) = \frac{\text{mean}(\text{abs}(\text{LODFs}))}{\text{std}(\text{abs}(\text{LODFs}))} \quad (3a)$$

$$M(i) = PF(i) \times \min\{\text{NLODF}(i), 1\} \quad (3b)$$

where NLODF and M are the normalized LODF metric and the proposed measure for selecting critical lines, respectively. $PF(i)$ is the power flow in line i during the normal operation; $\text{mean}(\cdot)$, $\text{std}(\cdot)$, and $\text{abs}(\cdot)$ indicate the mean, the standard deviation, and the absolute functions, respectively. In this article, we utilize PowerWorld Simulator [24] to calculate LODFs. However, if losing a line creates an islanding situation, PowerWorld Simulator does not calculate the exact LODF but assigns a large value to the remaining lines' LODF, which makes the $\text{NLODF}(i)$ value a very large number. Without an islanding situation, the $\text{NLODF}(i)$ values varies between $[0, 1]$.

The islanding situation is one of the worst contingencies that can occur in the system. Thus, the maximum NLODF value (i.e., 1) has been assigned to islanding situations. For handling islanding, we enforce the $\text{NLODF}(i)$ value corresponding to the islanding situation to be 1 as it is formulated in (3b). Assigning $\text{NLODF}=1$ to islanding situations prioritizes them to be considered previously all other situations. Equation (3b) represents the metric that is used to identify critical lines.

A metric that leverages the physics of the power networks as well as their topological information can effectively identify the critical lines in power systems. To this end, we first evaluate (3b) for all the lines in the network and select the first 10% of the lines with the highest value. Then, we search for the critical lines based on the search level, i.e., a prespecified parameter that determine how deep the proposed algorithm should search around the selected lines.

Fig. 1 visualizes the searching mechanism of the proposed approach for the IEEE 118-bus test case. We first select the lines with highest measure value of (3b). One of these lines is illustrated with green buses at both ends. The next step is finding all the nodes in graph G that are within distance d , i.e., search level, from these green nodes. For brevity, this set of buses is named "neighboring buses" in this article. For the $N - 1$ contingency analysis, (1) is applied to the neighboring buses, where s, t are different combination of buses in this set, and e is the line where both ends are shown by the green buses. For the $N - x$ contingency analysis, we should first determine E_G^i , the subset of edges of interest. In this connection, other lines with high value of (3b) in which their end buses are within the neighboring buses are selected for the $N - x$ contingency analysis. It is notable that there are two parameters that we can control to find the critical lines in the proposed approach. The first parameter is x , which determine the contingency analysis level. The second parameter is the "search level" that determines the number of neighboring buses that should be evaluated. A higher search level increases the search distance in the graph for the contingency analysis, i.e., more red buses in Fig. 1, but it incurs more computational burden. The execution time for

Algorithm 1 Discover critical branches in each sub graph

```

1. Initiate esa object and load case.
2. Extract Br= branch info, pf=branch power flow
3. Create Multigraph  $G(Br) = (V,E)$ .
4. LODF = CalculateLODF(case).
for b in Br do
   $LODFmetric_b = mean(LODF_b) / sd(LODF_b)$ 
  if  $LODFmetric_b \geq 10$  then
     $LODFmetric_b = abs(pf_b)$ 
  else
     $LODFmetric_b = LODFmetric_b * abs(pf_b)$ 
  end if
end for
5. SortedBranch = Sort b by  $LODFmetric_b$ 
6. InvgtdBranch=K% branches from SortedBranch
7. Initialize Invgtd_Br_Nbr
for from, to, cid, w in InvgtdBranch do
   $from_{sg} = subgraph(G, from, search\_level)$ 
   $to_{sg} = subgraph(G, to, search\_level)$ 
   $Inv\_Br\_Nbr = from_{sg}.nodes \cup to_{sg}.nodes$ 
   $Invgtd\_Br\_Nbr.append(Inv\_Br\_Nbr)$ 
end for
7. crit_lines_in_sg = Importance_Subgraph
  (Invgtd_Br_Nbr)
8. return crit_lines_in_sg

```

different search levels is elaborated for different test cases in Section IV.

In a cascading power transmission outage, component outages might propagate nonlocally; after one component outages, the next failure may be far, both topologically and geographically [31]. It is notable that the higher “*search level*” parameter enables the proposed approach to find critical lines that are far away from each other. However, higher “*search level*” incurs more computational complexity to the proposed approach, especially when it applies on larger test cases.

Another feature that enables the proposed approach to globally search for the critical lines is that it selects the first *Y*% of the lines (e.g., 10%) with higher measure value of (3b) and then starts searching around those lines. Those selected lines are usually distant, both topologically and geographically; that facilitates the proposed approach to find critical lines globally. The value for “*Y*” is computed by trial and error. A small value for *Y* does not select enough lines with high LODF values, and thus, several possible contingencies, that may be triggered by the nonselected lines with high LODFs, might not be evaluated by the proposed approach. Conversely, a large value for *Y* selects lines with moderate LODF values, and evaluating those lines incurs unwanted computational burden to the problem. Thus, we evaluated this tradeoff to find the best value for *Y* in this approach.

The steps of the proposed approach for identify critical lines are detailed in Algorithms 1 and 2.

Algorithm 2 Importance_Subgraph (*Investigated_Br_Nbr*)

```

Initialize imp_line_crit_sg_list
for ibn in Investigated_Br_Nbr do
   $inv\_sg = subgraph(ibn)$ 
  Initialize inv_sg_res dict
  for edge in inv_sg.edges do
     $inv\_sg\_res(edge) = LODF(edge)$ 
  end for
   $sorted\_inv\_sg\_res = sort(inv\_sg\_res)$ 
  Initialize crit_lines array.
  Initialize x in  $N - x$  contingency
  for key, val in  $sorted\_inv\_sg\_res[1 : x]$  do
     $from = val.from$ 
     $to = val.to$ 
     $crit\_lines.append(from, to)$ 
  end for
   $imp\_line\_crit\_sg\_cnt = 0$ 
  for line in crit_lines do
     $imp\_line\_cnt = 0$ 
    for fromn in inv_sg.nodes do
      for ton in inv_sg.nodes do
         $sp = srtst\_path(inv\_sg, from\_n, to\_n)$ 
        if line in sp then
           $imp\_line\_cnt++ = 1$ 
        end if
      end for
    end for
  end for
   $imp\_line\_crit\_sg\_cnt++ = imp\_line\_cnt/2$ 
end for
   $imp\_line\_crit\_sg\_list.append(imp\_line\_crit\_sg\_cnt)$ 
end for
return imp_line_crit_sg_list

```

IV. RESULTS AND ANALYSIS

This section demonstrates the effectiveness of the proposed approach using selected synthetic test cases from the benchmark library for electric power grids at Texas A&M University [32]. These test cases were selected since they mimic characteristics of real electric power grids. Synthetic electric grid cases are a representation of fictitious power grids that include detailed modeling of the power system elements [33]. Our implementations use PowerWorld [24] for contingency analysis, Python as a programming language, and ESA [34], a Python package that provides an easy to use and light-weight wrapper for interfacing with PowerWorld’s Simulator Automation Server, as an interference to communicate with PowerWorld. The results are computed using a laptop with an i7 1.80 GHz processor and 16 GB of RAM. Four different test cases including 200-, 500-, 2000-, and 2383-bus systems are investigated to authenticate the ability of the proposed approach in finding critical lines in electric power grids. The result for each test case is discussed in detail in sequel.

200-Bus Test System: The 200-bus test system is a synthetic test case with 245 branches and 49 generators that builds from

public information and a statistical analysis of real power systems [32]. In this connection, a brute-force search is performed for finding the $N - 1$ contingencies in the 200-bus test system to evaluate the performance of the proposed algorithm. It is notable that the 200-bus test system is $N - 1$ resilient, and as it was expected, the brute-force search finds only one reserved limit violation, where there is not enough active power reserves in the make-up power specification to cover the active power changes by the contingency. The application of the proposed approach finds the exact same violation for the $N - 1$ contingency analysis. This verifies the accuracy of the proposed approach for identifying contingencies in power systems. To further evaluate the ability of the proposed approach in finding the critical lines, another brute-force search is fulfilled for identifying $N - 2$ contingencies in the 200-bus test system, and the obtained results match those obtained by the proposed algorithm. The brute-force search and the proposed approach find the $N - 2$ contingencies for the 200-bus test cases in 230 and 38 s, respectively. Comparing the computational times of the brute force search and the proposed approach for finding $N - 2$ contingencies reveals the ability of the proposed approach in finding critical lines in large test cases, which cannot be done by the exhaustive search. The proposed approach is applied to identify critical lines for different contingency analyses in the 200-bus test system. The results are summarized in Table II. The first column lists the x in the $N - x$ contingency term. The second and the third columns represent critical lines and contingency types, respectively, and the fourth column represents number of contingencies.

With the contingency analysis tool in PowerWorld, we capture four types of “limit violations” in this article, including reserve limit, overflow, undervoltage, and unsolved. The unsolved cases represent a situation where there is no solution for the power flow equations or they cannot converge. The overflow and undervoltage violations can be counted by the number of components that fall into the category. The critical lines in the second column are those that their lost has severe impact on the network performance. These data are crucial for power system operation and planning. The one-line diagram of the 200-bus test cases and the corresponding violations caused by the outage of lines [189, 187], [187, 121] are illustrated in Fig. 3. The bottom portion of this figure shows the zoom-in view of the area that violation has occurred.

The box-and-whisker plot shown in Fig. 4 compares the impact of x on the execution times of $N - x$ contingency analysis for 200-bus test case. Every box contains the execution time of various contingency analyses with different search levels (i.e., 1–8) and a specific x in the $N - x$ term. The lower and upper ends of the boxes in Fig. 4 reflect the first and third quartiles, and the lines inside the boxes denote the median. There is not a significant difference between the median lines of the boxes, which reflects the fact that the proposed approach can solve higher-order $N - x$ contingency analyses within a reasonable time. This result is expected because for a same search level, a same set of neighboring buses are utilized in identifying different $N - x$ contingency analyses. In other words, the execution time of different $N - x$ contingency analyses for specific search levels changes slightly for different x values. The plot in Fig. 4

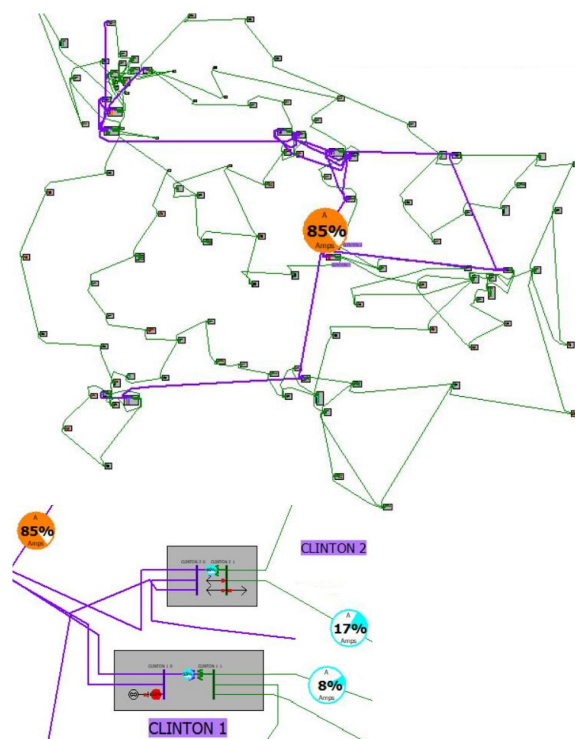


Fig. 3. One-line diagram of the 200-bus test case and the corresponding overflow contingency caused by the removal of [136, 133], [135, 133] lines. The bottom portion of Fig. 3 shows the zoom-in view of the area that violation has occurred.

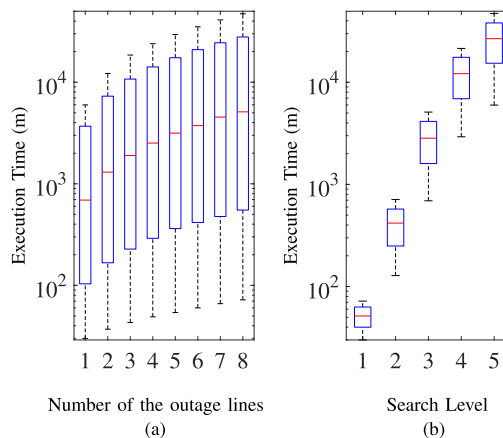


Fig. 4. (a) Execution time comparisons of various contingency analysis using generalized contingency analysis. (b) Execution time comparisons of different search levels in generalized contingency analysis for 200-bus test system.

shows the impact of different search levels on the execution times of the contingency analyses in the 200-bus test system. The execution time linearly increases with the search level increment. The increase in the search level enables the proposed approach to search in a broader area but incurs computational burden.

500-Bus Test System: The 500-bus test system, with 597 branches and 90 generators, is a relatively large synthetic test case that is used for evaluating the ability of the proposed approach in identifying critical lines. All the synthetic test cases

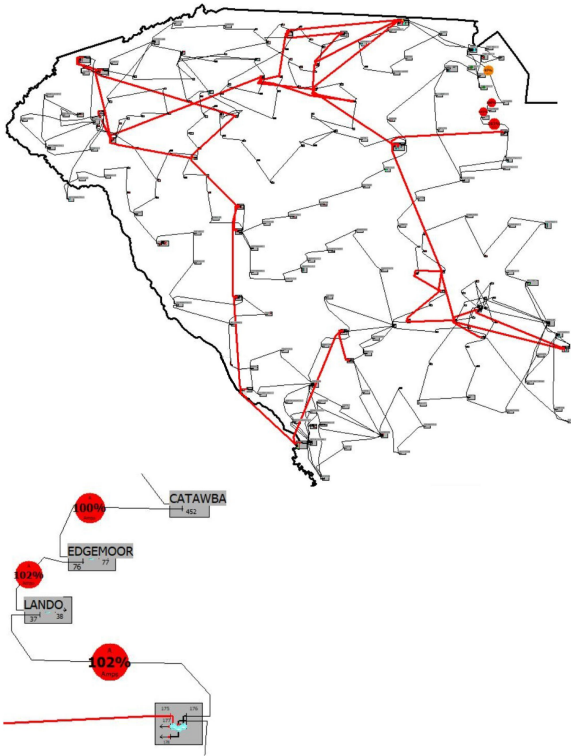


Fig. 5. One-line diagram of the 500-bus test cases and the corresponding violations caused by the removal of [142, 141], [87, 141], [424, 423] lines. The bottom portion of Fig. 5 shows the zoom-in view of the area that violation has occurred.

used in this article are $N - 1$ resilient, i.e., no contingency occurs with the outage of a single line. Furthermore, these test cases are resilient enough that it is hard to identify a contingency by randomly removing multiple lines. However, the proposed algorithm can identify overflow violations, which occur by the outage of three branches, i.e., [142, 141], [424, 423], [87, 141]. Finding contingencies caused by the outage of three lines reveals the ability of the proposed approach in identifying critical lines in relatively large resilient test cases such as the 500-bus test system.

The results of different contingency analyses for finding the most critical lines in the 500-bus test system are summarized in Table II. It is notable that multiple critical lines are identified for each $N - x$ contingency analysis. The results in Table II summarize both the contingency type and number of limit violations for the outage of critical lines in the 500-bus test system.

The execution time of the contingency analysis approaches plays an important role in power system operation and control. Many approaches for identifying critical lines are not tractable to be applied on large test cases. However, the proposed approach performs the $N - 3$ contingency analysis in the relatively large 500-bus test case in just 5 min. when the search level is set to 3. The execution time linearly increases with the search level. The one-line diagram of the 500-bus test case and the corresponding violations caused by the outage of the lines [142, 141], [424, 423], [87, 141] are illustrated in Fig. 5. The bottom portion of

TABLE I
RESULTS FROM APPLYING THE PROPOSED APPROACH TO
200-BUS TEST SYSTEM

x	Critical Lines	Violation Type	Number of Violations
1	[189, 187]	Reserve limit	NA
2	[189, 187],[187, 121]	Reserve limit	NA
2	[136, 133],[135, 133]	Overflow	1
2	[189, 187],[136, 133]	Overflow	1
3	[189, 187],[187, 121],[188, 187]	Reserve Limit	NA
3	[189, 187],[187, 121],[154, 149]	Reserve Limit	NA
3	[136, 133],[135, 133],[125, 123]	Overflow Undervoltage	2 18
3	[189, 187],[136, 133],[135, 133]	Unsolved	NA
4	[189, 187],[187, 121],[188, 187],[179, 178]	Undervoltage	25
4	[189, 187],[187, 121],[154, 149],[152, 149]	Overflow	2
4	[136, 133],[135, 133],[125, 123],[126, 123]	Overflow Undervoltage	2 84
5	[189, 187],[187, 121],[188, 187],[179, 178],[45, 187]	Undervoltage	29
5	[189, 187],[187, 121],[154, 149],[152, 149],[153, 149]	Unsolved	NA
5	[136,133],[135, 133],[125, 123],[126, 123],[127, 123]	Unsolved	NA
5	[125, 123],[126, 123],[127, 123],[124, 123],[134, 133]	Overflow Undervoltage	1 17
6	[189, 187],[187, 121],[188, 187],[179, 178],[45, 187],[188, 89]	Undervoltage	26
6	[136,133],[135, 133],[125, 123],[126, 123],[127, 123],[188, 89]	Unsolved	NA
6	[189, 187],[187, 121],[154, 149],[152, 149],[153, 149],[155, 149]	Unsolved	NA
7	[136, 133],[135, 133],[125, 123],[126, 123],[127, 123],[124, 123],[134, 133]	Overflow Undervoltage	2 183
7	[189, 187],[187, 121],[188, 187],[179, 178],[45, 187],[188, 89],[121, 178]	Undervoltage	36
7	[189, 187],[187, 121],[154, 149],[152, 149],[153, 149],[155, 149],[16, 15]	Unsolved	NA
8	[136, 133],[135, 133],[125, 123],[126, 123],[127, 123],[124, 123],[134, 133],[128, 133]	Unsolved	NA
8	[136, 133],[135, 133],[125, 123],[126, 123],[127, 123],[124, 123],[134, 133],[30, 29]	Overflow Undervoltage	2 151
8	[189, 187],[187, 121],[154, 149],[152, 149],[153, 149],[155, 149],[16, 15],[188,187]	Unsolved	NA
8	[189, 187],[187, 121],[188,187],[45, 187],[188,89],[121,178],[14, 15],[188,187]	Unsolved	NA

Fig. 5 depicts the zoomed-in view of the area where the violation has happened (i.e., the 112% overflow in the line).

The plot in Fig. 6(a) compares the impact of x on the execution time of $N - x$ contingency analysis for the 500-bus test case. As it was expected, the execution time of $N - x$ contingency analysis for different x values does not change significantly for

TABLE II
RESULTS FROM APPLYING THE PROPOSED APPROACH TO
500-BUS TEST SYSTEM

x	Critical Lines	Violation Type	Number of Violations
3	[7099, 7095],[7098, 7095],[7058, 7095]	Reserve Limit	NA
4	[5262, 5260],[5263, 5260],[5317, 5260],[5358, 5179]	Overflow	4
4	[7099, 7095],[7098, 7095],[7058, 7095],[6161, 7018]	Reserve Limit	NA
4	[7099, 7095],[7098, 7095],[7058, 7095],[7311, 7304]	Reserve Limit	NA
5	[5262, 5260], [5263, 5260], [5317, 5260], [5358, 5179], [5260, 5045]	Overflow	6
5	[5262, 5260], [5263, 5260], [5317, 5260], [5358, 5179], [5206, 5204]	Overflow	4
5	[7099, 7095],[7098, 7095],[7058, 7095],[6161, 7018],[7311, 7304]	Reserve Limit	NA
5	[7099, 7095],[7098, 7095],[7058, 7095],[7346, 7125],[7197, 7186]	Reserve Limit	NA
5	[5360, 5358], [8071, 8067], [5317, 5260], [5358, 5179],[5179, 5380]	Overflow	1
6	[5362, 5260], [5363, 5260], [5317, 5260], [5358, 5179],[5260, 5045],[5179, 5380]	Overflow	6
6	[5362, 5260], [5363, 5260], [5317, 5260], [5358, 5179],[5206, 5204],[5207, 5204]	Overflow	4
6	[7099, 7095], [7098, 7095], [7058, 7095], [6161, 7018],[7311, 7304],[7046, 7042]	Reserve Limit	NA
6	[7099, 7095], [7098, 7095], [7058, 7095], [7346, 7125],[7197, 7186],[7196, 7186]	Reserve Limit	NA
7	[5262, 5260], [5263, 5260], [5360, 5358], [5303, 5295], [5302, 5295], [5317, 5260], [5358, 5179]	Overflow Undervoltage	5 14
7	[5262, 5260],[5263, 5260],[5317, 5260],[5358, 5179], [5260, 5045],[5179, 5380], [5380, 5384]	Overflow	6
7	[5262, 5260],[5263, 5260],[8071, 8067],[5068, 5063], [5067, 5063],[5069, 5063], [5317, 5260]	Overflow	1
7	[5262, 5260],[5263, 5260],[5317, 5260],[5358, 5179],[5206, 5204],[5207, 5204], [5208, 5204]	Overflow	5
7	[7099, 7095], [7098, 7095], [7058, 7095], [6161, 7018],[7311, 7304],[7046, 7042], [7045, 7042]	Reserve Limit	NA
7	[7099, 7095], [7098, 7095], [7058, 7095], [7311, 7304],[7046, 7042],[7045, 7042], [7304, 7095]	Reserve Limit	NA
8	[5262, 5260],[5263, 5260],[5360, 5358],[5303, 5295],[5302, 5295],[5317, 5260],[5358, 5179],[5260, 5045]	Overflow Undervoltage	5 14
8	[5262, 5260],[5263, 5260],[5317, 5260],[5358, 5179],[5206, 5204],[5207, 5204], [5208, 5204],[5260, 5045]	Overflow	5
8	[5262, 5260],[5263, 5260],[5317, 5260],[5358, 5179],[5260, 5045],[5179, 5380], [5380, 5384],[5047, 5260]	Overflow	6
8	[5262, 5260],[5263, 5260],[8071, 8067],[5068, 5063], [5067, 5063],[5069, 5063], [5317, 5260],[5358, 5179]	Unsolved	NA
8	[6146, 6141],[6145, 6141],[6246, 6239],[6245, 6239],[6244, 6239],[6147, 6141],[6149, 6141],[6161, 7018]	Overflow	1
8	[7099, 7095], [7098, 7095], [7058, 7095], [6161, 7018],[7311, 7304],[7046, 7042], [7045, 7042],[6239, 7414]	Reserve Limit	NA

TABLE III
RESULTS FROM APPLYING THE PROPOSED APPROACH TO
2000-BUS TEST SYSTEM

x	Critical Lines	Violation Type	Number of Violations
1	[163, 165]	Overflow Undervoltage	21 21
1	[74, 76]	Overflow Undervoltage	17 17
2	[163, 165], [2187, 165]	Overflow Undervoltage	24 25
2	[73, 75], [16, 18]	Overflow Undervoltage	23 19
3	[163, 165], [2187, 165], [2188, 163]	Overflow Undervoltage	24 25
3	[73, 75], [16, 18], [70, 73]	Overflow Undervoltage	23 21
4	[163, 165], [2187, 165], [2188, 163], [163, 172]	Overflow Undervoltage	23 26
4	[73, 75], [16, 18], [70, 73], [16, 73]	Overflow Undervoltage	23 21
5	[163, 165], [2187, 165], [2188, 163], [163, 172], [164, 163]	Overflow Undervoltage	23 26
5	[73, 75], [16, 18], [70, 73], [16, 73], [87, 73]	Overflow Undervoltage	23 21
6	[163, 165], [2187, 165], [2188, 163], [163, 172], [164, 163], [15, 165]	Unsolved	NA
6	[73, 75], [16, 18], [70, 73], [16, 73], [87, 73], [75, 18]	Overflow Undervoltage	23 22
7	[163, 165], [2187, 165], [2188, 163], [163, 172], [164, 163], [15, 165], [159, 165]	Unsolved	NA
8	[74, 76], [17, 18], [76, 105], [17, 74], [27, 74], [74, 108], [74, 56], [75, 18]	Overflow Undervoltage	23 24
8	[73, 75], [16, 18], [70, 73], [16, 73], [87, 73], [75, 18], [76, 75], [18, 76]	Overflow Undervoltage	27 29

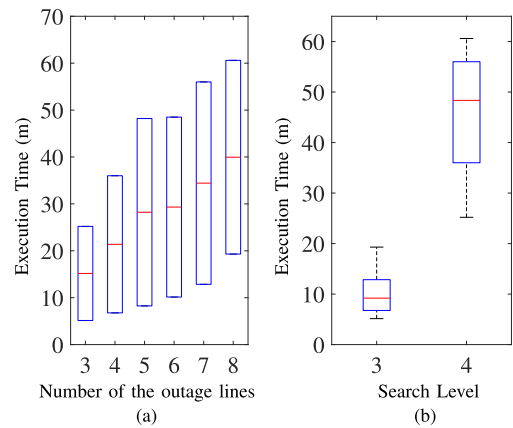


Fig. 6. (a) Execution time comparisons of various contingency analysis using generalized contingency analysis. (b) Execution time comparisons of different search levels in generalized contingency analysis for 500-bus test system.

a same “search level.” Similarly, the plot in Fig. 6(b) shows the impact of different search levels on the execution time of $N - x$ contingency analysis for the 500-bus test system. Although the increment in search level increases the execution time of

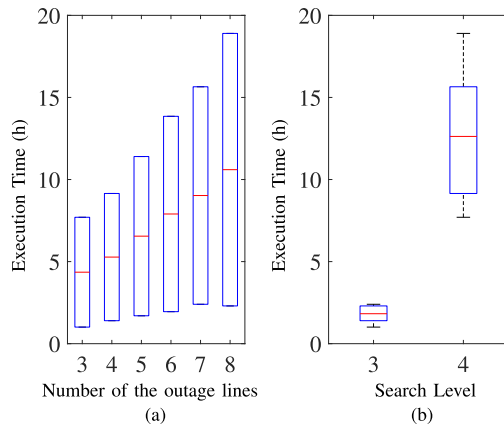


Fig. 7. (a) Execution time comparisons of various contingency analysis using generalized contingency analysis. (b) Execution time comparisons of different search levels in generalized contingency analysis for 2000-bus test system.

$N - x$ contingency analysis, the proposed algorithm remains computationally tractable for higher search levels.

2000-Bus Test System: The 2000-bus test system, with 3206 branches and 544 generators, is a synthetic test case that is utilized for evaluating the ability of the proposed approach in identifying critical lines in large test systems. The size of this synthetic test system makes it a challenging test case for contingency analysis, because the number of combinatorial scenarios that need to be evaluated increases drastically by the size of the network. Thus, for this test case, the exhaustive search approaches are not tractable even for the $N - 3$ contingency analysis. Like other synthetic test cases investigated in this article, the 2000-bus test case is $N - 1$ resilient. Furthermore, this test system is resilient enough that finding a contingency that causes a violation even after multiple lines are outaged, e.g., randomly removing 7 lines ($N - 7$ contingency), is not easy. Conversely, the proposed approach can easily find a $N - 4$ contingency caused by the outages of four lines including [5262, 5260], [5263, 5260], [5317, 5260], [5358, 5179].

The impact of x and search level on the execution time of $N - x$ contingency analysis for 2000-bus test case are shown in Fig. 7(a) and (b), respectively. For the same search level, execution times of the $N - x$ contingency analyses in Fig. 7(a) change linearly for different x values. This enables the proposed approach to evaluate higher orders of $N - x$ contingency analysis in large test systems. The lower search level mirrors the lower number of lines that need to be investigated. Thus, for different search levels in Fig. 7(b), the lower search level value yields the faster $N - x$ contingency analysis.

Polish 2383-Bus Test System: The Polish 2383-bus test system, with 2383 buses, 2896 branches, and 327 generators, is the largest test case in this article that is used for evaluating the ability of the proposed approach in identifying critical lines. The exhaustive search approach is not tractable for performing contingency analysis for $x \geq 2$ in large test systems since the number of combinatorial scenarios that need to be evaluated soars by the number of branches. Thus, the obtained results for this test systems cannot be compared with the exhaustive search approach for $x \geq 2$. Unlike the other test systems investigated

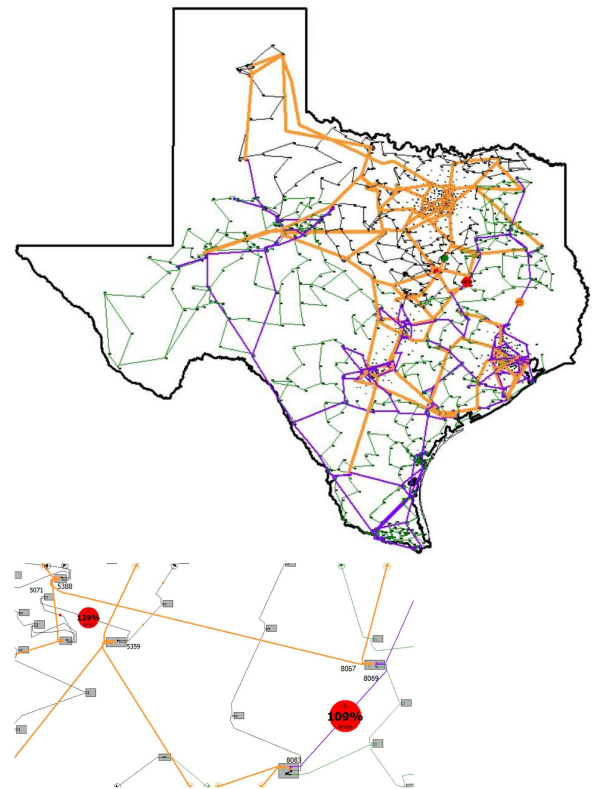


Fig. 8. One-line diagram of the 2000-bus test cases and the corresponding overflow contingency caused by the removal of [5262, 5260], [5263, 5260], [5317, 5260], [5358, 5179] lines. The bottom portion of Fig. 8 shows the zoom-in view of the area that violation has occurred.

TABLE IV
RESULTS FROM APPLYING THE PROPOSED APPROACH TO THE POLISH
2383-BUS TEST SYSTEM

Test Cases	Brute-Force Search			BC Approach in [6]			Proposed Approach		
	N-1	N-2	N-3	N-1	N-2	N-3	N-1	N-2	N-3
200-Bus	✓	✓	X	✓	NA	NA	✓	✓	✓
500-Bus	✓	✓	X	✓	NA	NA	✓	✓	✓
2000-Bus	✓	X	X	X	NA	NA	✓	✓	✓
Polish 2383-Bus	✓	X	X	X	NA	NA	✓	✓	✓

✓: Tractable, X: Not tractable, NA: Not Applicable.

in this article, the Polish test system is not $N - 1$ resilient and more than 1000 $N - 1$ contingencies are found by the exhaustive search approach. Note that the proposed algorithm finds a portion of critical lines found by the exhaustive search since it is focused on the first $Y\%$ of lines with high LODF values. However, the critical lines found by the proposed algorithm are those that their outages cause severe violations in the system and, thus, are the priority of system operator to find them.

The critical lines identified by the proposed approach are tabulated in Table IV for the Polish test system. Note that these lines are selected from a large pool of critical lines identified by the proposed approach since their outages have more severe impact on the system compared to other critical lines in the pool. Fig. 9 visualizes the impact of the search level and the order of contingency analysis on the computational time of the proposed algorithm. The computational time of the proposed algorithm

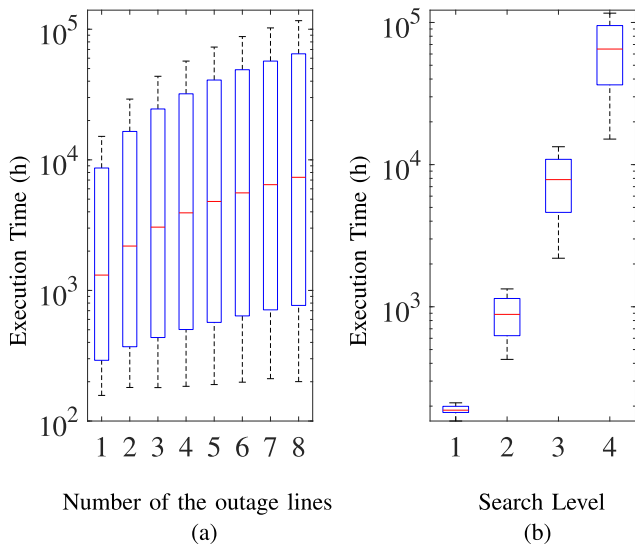


Fig. 9. (a) Execution time comparisons of various contingency analysis using generalized contingency analysis. (b) Execution time comparisons of different search levels in generalized contingency analysis for 2383-bus test system.

TABLE V
COMPARING THE TRACTABILITY OF DIFFERENT APPROACHES FOR THE $N - X$ CONTINGENCY ANALYSIS

Test Cases	Brute-Force Search			BC Approach in [6]			Proposed Approach		
	N-1	N-2	N-3	N-1	N-2	N-3	N-1	N-2	N-3
200-Bus	✓	✓	X	✓	NA	NA	✓	✓	✓
500-Bus	✓	✓	X	✓	NA	NA	✓	✓	✓
2000-Bus	✓	X	X	X	NA	NA	✓	✓	✓
Polish 2383-Bus	✓	X	X	X	NA	NA	✓	✓	✓

✓: Tractable, X: Not tractable, NA: Not Applicable.

linearly increases by the search level and the contingency analysis order.

VI. DISCUSSION AND ACHIEVEMENTS

The numerical results in Section IV validate the ability of the proposed algorithm in identifying critical lines using several synthetic power systems. The group betweenness centrality metric enables the proposed approach to identify multiple critical lines at the same time. Leveraging the LODF metric in addition to the group betweenness centrality, which is the core difference between the proposed approach and those in the literature, enables the proposed approach to perform the $N - x$ contingency analyses for large test cases. For instance, even $N - 1$ contingency analysis can be computationally intensive for large cases, where the LODF metric is particularly useful. Incorporating the LODF metric enables the proposed approach to be applied on larger test systems by shrinking the search space of the problem via selecting the lines that their outages have severe impact on the system. This is evident by comparing the fifth and eighth columns in Table V. Performing the $N - 2$ contingency analysis on the 2000- and 2383-bus tests systems by the brute force search method requires evaluating 2 748 340 and 3 711 450 contingencies, respectively, which is not tractable. Leveraging the LODF metric, the proposed method quickly focuses on the lines with high LODF values for finding the

critical lines and, thus, make the $N - x$ contingency analyses tractable for $x \geq 2$ in large test cases.

The proposed approach finds critical lines in the resilient test cases such as the 2000-bus test case where randomly removing multiple lines does not cause any violations in the system. To further discuss the ability of the proposed approach for identifying critical lines in the resilient test cases, we randomly remove 7 lines from the 2000-bus test system to trigger a violation in the system. We tried this several times, but no violation was recognized in these trials. However, the proposed approach was able to find a result from $N - 4$ contingency analysis in the same system. This validates the ability of the proposed approach for identifying critical lines irrespective of the resilience level of the test cases. Moreover, both the brute force method and the proposed algorithm find [189 187] line as the only critical line in 200-bus test system that its outage causes a reserve limit to be reached in the system. However, the pure BC approach in [6] identifies [1 119] as a critical line while its outages does not cause any violation. Thus, incorporating LODFs not only makes the proposed approach faster but also makes the approach more accurate in identifying critical lines.

Note that the pure BC approach in [6] only find one critical line while the proposed approach identifies a set of critical lines for the $N - x$ contingency analysis. The proposed approach enables power system operators and planners to identify multiple critical lines in the large systems, that their outages have severe impact on the system, by performing the $N - x$ contingency analysis for $x \geq 2$, which is not tractable for other contingency analysis approaches in the literature, e.g., [2] and [6].

Contingency analysis is fundamentally a *preview* analysis tool. It simulates and quantifies the results of failures that could occur in the power system in the immediate future. Contingency analysis is either used as a study tool for the offline analysis of contingency events, or as an online tool to show power system operators what would be the effects of future outages. This allows operators to be better prepared to react to outages by using preplanned recovery scenarios.

By analyzing the effects of contingency events in advance, problems and unstable situations can be identified, critical configurations can be recognized, operating constraints and limits can be applied, and corrective actions can be planned. The contingency analysis can be used for scheduling the withdrawal of power system equipment for periodic or restorative maintenance in which the schedule for planned outages is arranged for minimal risk of problems by using the contingency analysis studies, to avoid scheduling concurrent outages of critical system elements.

Typically, the power system model is tested for many hundreds of possible problems, including the failure of each generator and line, as well as other elements. These events are placed on the contingency list by experienced planning and operations engineers because of their importance—the severity of their effects, and their likelihood (probability) of occurrence. The proposed approach in this article can be used to quickly identify critical components and thus establish the contingency lists for large test cases. The proposed approach for contingency analysis

can be used not only as a system planning tool but is *fast enough* to be used as an online analysis tool by power system operator, to support preventive and corrective operator actions in case of problems.

VII. CONCLUSION

The proposed approach utilizes the physical and topological characteristics of electric power grids for identifying the critical lines in $N - x$ contingency analysis. Augmenting the LODF metric that captures the physics of the electric grid, with the group betweenness centrality metric that captures the topology of the electric grid, enables the proposed approach to effectively find the most critical lines, even in large systems. Protecting the resulting elements can enable power grid operators to prevent cascading failures and operate the system more reliably. The curse of dimensionality inherent to generalized contingency analysis (i.e., $N - x$ contingency analysis with $x \geq 2$) makes this problem computationally infeasible, for even midsize electric grids, when solved by the traditional approaches. The proposed approach decouples the computation from the problem size, which enables it to perform the $N - x$ contingency analysis with $x \geq 2$ in a reasonable time irrespective of the grid size. Results show that the proposed approach acts as a straightforward and computationally tractable search engine that can quickly identify critical lines even in large cases. Our ongoing work aims to improve our approach by making it as independent as possible from “search level.” In other words, we are working on an approach that can search in different parts of electric power grids at the same time. Other ongoing work is developing further improvements to the proposed approach by updating the LODF metric after each line outage.

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